1 In an arithmetic series, the 6th term is 39
In the same arithmetic series, the 19th term is 7.8

Work out the sum of the first 25 terms of the arithmetic series.

$T_n = a + (n-1)d$

$T_6 = 39 = a + 5d$ — ①

$T_{19} = 7.8 = a + 18d$ — ②

Substitute ① into ② :

$7.8 = (39 - 5d) + 18d$

$7.8 - 39 = 13d$

$-31.2 = 13d$ ①

$d = \frac{-31.2}{13} = -2.4$

Substitute $d = -2.4$ into ①

$39 = a + 5(-2.4)$

$39 = a - 12$

$a = 39 + 12$

$= 51$ ①

Sum of first 25 term : $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{25} = \frac{25}{2}\left[2(51) + (24)(-2.4)\right]$ ①

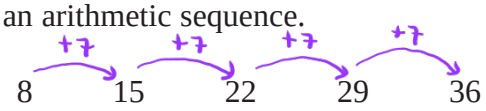
$= 12.5(102 - 57.6)$

$= 555$ ①

555

(Total for Question 1 is 4 marks)

2 Here are the first five terms of an arithmetic sequence.



Work out the sum of all the terms from the 50th term to the 100th term inclusive.

$a = 8$ common difference, $d = 7$ (1)

$S_{100} = \frac{100}{2}\left[2(8) + (100-1)7\right]$

$= 50\,(16 + 693)$

$= 35\,450$

$S_{50} = \frac{50}{2}\left[2(8) + (50-1)7\right]$ (1)

$= 25\,(16 + 343)$

$= 8975$

$S_{100} - S_{50} = 35\,450 - 8975$

$= 26\,475$

since we also need to include the 50th term :

$26\,475 + (T_{50} = 8 + 49(7))$

$= 26\,475 + 351$ (1)

$= 26\,826$ (1)

Alternatively, we can just work out $S_{100} - S_{49}$, as this will also include T_{50} in it.

26 826

(Total for Question 2 is 4 marks)

3 The first term of an arithmetic series S is –6
The sum of the first 30 terms of S is 2865

Find the 9th term of S.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a + (n-1)d$

$a = -6$

$S_{30} = 2865 = \frac{30}{2}[2(-6) + (30-1)d]$ (1)

$2865 = 15(-12 + 29d)$

$191 = -12 + 29d$

$29d = 203$

$d = 7$ (1)

$T_9 = -6 + (9-1)7$

$= -6 + 8(7)$ (1)

$= -6 + 56$

$= 50$ (1)

50

(Total for Question 3 is 4 marks)

4 Mario is going to save \$50 in the year 2021

He is going to continue to save, up to and including the year 2070, by increasing the amount he saves each year by \$$k$

Mario will save a total of \$33 125 from 2021 to 2070

Work out the value of k.

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$n = 50$ (1) $a = 50$ $d = k$

$33\,125 = \frac{50}{2}\left[2(50) + (49)k\right]$

$33\,125 = 25\left(100 + 49k\right)$ (1)

$\frac{33\,125}{25} = 100 + 49k$

$1325 - 100 = 49k$

$49k = 1225$

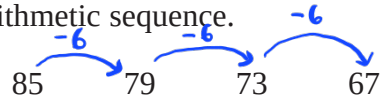
$k = \frac{1225}{49}$

$= 25$ (1)

$k =$ 25

(Total for Question 4 is 3 marks)

5 Here are the first 4 terms of an arithmetic sequence.



Find an expression, in terms of n, for the nth term of the sequence.

Common difference, $d = -6$

first term, $a = 85$

$T_n = a + (n-1)d$

$T_n = 85 + (n-1)(-6)$ (1)

$= 85 - 6n + 6$

$= 91 - 6n$ (1)

$91 - 6n$

(Total for Question 5 is 2 marks)

6 The sum of the first N terms of an arithmetic series, S, is 292
The 2nd term of S is 8.5
The 5th term of S is 13

Find the value of N.
Show clear algebraic working.

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = a + (n-1)d$

$8.5 = a + (2-1)d$ $\qquad$ $13 = a + (5-1)d$

$8.5 = a + d$ - (1) $\qquad$ $13 = a + 4d$ - (2)

(1)

substitute (1) into (2)

$13 = (8.5 - d) + 4d$

$13 = 8.5 + 3d$

$3d = 4.5$

$\therefore d = 1.5$

$\therefore a = 8.5 - 1.5$

$= 7$ (1)

$S_N = 292 = \frac{N}{2}\left[2(7) + (N-1)1.5\right]$

$584 = N(14 + 1.5N - 1.5)$

$584 = 1.5N^2 + 12.5N$

$1.5N^2 + 12.5N - 584 = 0$ (1)

$3N^2 + 25N - 1168 = 0$

$N = \frac{-25 \pm \sqrt{25^2 - 4(3)(-1168)}}{2(3)}$ (1)

$= \frac{-25 \pm \sqrt{14641}}{6}$

$= \frac{-25 \pm 121}{6}$ $= 16$ or -24.3.

$N = 16$ (1) ↳ positive integers

$N =$ 16

(Total for Question 6 is 5 marks)

7 The nth term of an arithmetic series is u_n where $u_n > 0$ for all n
The sum to n terms of the series is S_n

Given that $u_4 = 6$ and that $S_{11} = (u_6)^2 + 18$

find the value of u_{20}

$u_4 = 6 = a + (4-1)d$

$6 = a + 3d$

$a = 6 - 3d$ (1)

$S_{11} = \frac{11}{2}[2a + 10d] = (a+5d)^2 + 18$ (1)

Substitute $a = 6 - 3d$ into equation of S_{11} :

$\frac{11}{2}[2(6-3d) + 10d] = (6 - 3d + 5d)^2 + 18$ (1)

$\frac{11}{2}(12 - 6d + 10d) = (6 + 2d)^2 + 18$

$\frac{11}{2}(12 + 4d) = 36 + 24d + 4d^2 + 18$

$66 + 22d = 54 + 24d + 4d^2$

$4d^2 + 24d - 22d + 54 - 66 = 0$

$4d^2 + 2d - 12 = 0$

$2d^2 + d - 6 = 0$ (1)

$(2d - 3)(d + 2) = 0$

$d = \frac{3}{2}$ or $d = -2$

since $u_n > 0$, $d = -2$ is not valid.

$d = \frac{3}{2}$ and $a = \frac{3}{2}$ (1)

$u_{20} = \frac{3}{2} + (19)(\frac{3}{2})$

$= 30$ (1)

30

(Total for Question 7 is 6 marks)

8 An arithmetic series has first term a and common difference d.

The sum of the first $2n$ terms of the series is four times the sum of the first n terms of the series.

Find an expression for a in terms of d.
Show your working clearly.

$S_{2n} = \frac{2n}{2}\left[2a + (2n-1)d\right]$

$= n(2a + 2nd - d)$ (1)

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}(2a + nd - d)$

$S_{2n} = 4S_n$

$n(2a + 2nd - d) = 4\left[\frac{n}{2}(2a + nd - d)\right]$ (1)

$n(2a + 2nd - d) = 2n(2a + nd - d)$

$2a + 2nd - d = 2(2a + nd - d)$

$2a + 2nd - d = 4a + 2nd - 2d$

$4a - 2a = 2nd - 2nd - d + 2d$ (1)

$2a = d$

$a = \frac{d}{2}$ (1)

$a =$ $\frac{d}{2}$

(Total for Question 8 is 4 marks)

9 In a warehouse there are two types of shelves, type **R** and type **S**.

These two types of shelves are arranged into shelving units that form a sequence of patterns.

Here are the first three terms in the sequence.

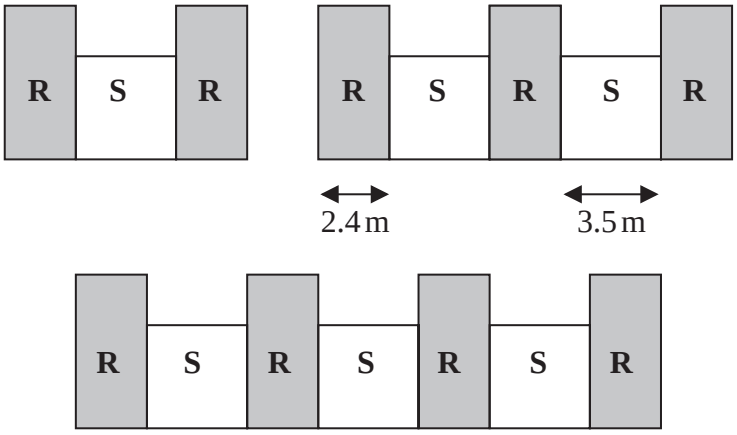


Diagram **NOT** accurately drawn

2R + 1S

The width of each type **R** shelf is 2.4 m and the width of each type **S** shelf is 3.5 m

(a) Work out the total width of a shelving unit that has 6 type **R** shelves.

[6 R shelves + 5 S shelves]

∴ 6 × R + 5 × S

= (6 × 2.4) + (5 × 3.5) (1)

= 14.4 + 17.5

= 31.9 (1)

31.9 m

(2)

A shelving unit has n type **R** shelves.
The total width of this shelving unit is W metres.

(b) Find an expression for W in terms of n
Give your answer in its simplest form.

$T_1 = 2R + S$

$T_2 = 3R + 2S$

$T_n = nR + (n-1)S$

$W = n(2.4) + (n-1)(3.5)$

$= 2.4n + 3.5n - 3.5$ (1)

$W = 5.9n - 3.5$ (1)

$W =$ 5.9n - 3.5

(2)

(Total for Question 9 is 4 marks)

10 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[2a+(10-1)d]$

$= 10a + 45d$

$S_5 = \frac{5}{2}[2a+(5-1)d]$

$= 5a + 10d$

$S_{10} = 4 \times S_5$

$10a + 45d = 4(5a + 10d)$ (1)

$10a + 45d = 20a + 40d$

$10a = 5d$

$d = 2a$ — (1) (1)

$T_n = a + (n-1)d$

$T_8 = 45 = a + (8-1)d$

$45 = a + 7d$ — (2) (1)

Substitute (1) into (2):

$45 = a + 7(2a)$ (1)

$45 = 15a$

$a = 3$ (1)

∴ First term, $a = 3$.

3

(Total for Question 10 is 5 marks)

11 Here are the first five terms of an arithmetic sequence.

1 5 9 13 17

(a) Find an expression, in terms of n, for the nth term of this sequence.

$d = 4$

$a = 1$

$T_n = a + (n-1)d$

$= 1 + (n-1)4$ (1)

$= 1 + 4n - 4$

$= 4n - 3$ (1)

$4n - 3$

(2)

The nth term of another arithmetic sequence is $3n + 5$

(b) Find an expression, in terms of m, for the $(2m)$th term of this sequence.

$= 3(2m) + 5$

$= 6m + 5$ (1)

$6m + 5$

(1)

(Total for Question 11 is 3 marks)

12 A polygon has n sides, where $n > 5$

When arranged in order of size, starting with the largest number, the sizes of the interior angles of the polygon, in degrees, are the terms of an arithmetic sequence.

Here are the first five terms of this sequence.

177 175 173 171 169

Find the value of n
Show clear algebraic working.

$d = -2$ (1)

$S_n = \frac{n}{2}\left[2(177) + (n-1)(-2)\right]$

$= n(178 - n)$ (1)

$n(178 - n) = (n-2) \times 180$ (1)

$178n - n^2 = 180n - 360$

$n^2 + 2n - 360 = 0$ (1)

$(n-18)(n+20) = 0$

(1)

$n = 18$

(1)

$n =$ 18

(Total for Question 12 is 6 marks)

13 An arithmetic series has first term 1 and common difference 4

Find the sum of all terms of the series from the 41st term to the 100th term inclusive.

$S_{100} = \frac{100}{2} \times 2 + 99(4)$

$= 50 \times 398$

$= 19\,900$ (1)

$S_{40} = \frac{40}{2} \times 2 + 39(4)$

$= 20 \times 158$

$= 3160$ (1)

$S_{100} - S_{40} = 19900 - 3160$ (1)

$= 16\,740$ (1)

16 740

(Total for Question 13 is 4 marks)

14 An arithmetic series has first term a and common difference d, where d is a prime number.

The sum of the first n terms of the series is S_n and

$S_m = 39$

$S_{2m} = 320$

Find the value of d and the value of m
Show clear algebraic working.

$S_m = \frac{m}{2}[2a + (m-1)d] = 39$ (1)

$2am + m^2d - md = 78$ – (1)

$S_{2m} = \frac{2m}{2}[2a + (2m-1)d] = 320$ (1)

$2am + 2m^2d - md = 320$ – (2)

(2) – (1) :

$m^2d = 320 - 78$

$m^2d = 242$ (1)

if $d = 2$: $m^2 = \frac{242}{2}$ (1)

$m^2 = 121$

$m = 11$

$d =$ 2 (1)

$m =$ 11

(Total for Question 14 is 5 marks)

15 An arithmetic sequence has first term 8 and common difference 11
The sequence has k terms, where $k > 21$

The sum of the last 20 terms of the sequence is 10 170

Find the value of k
Show clear algebraic working.

$S_{20} = \frac{\cancel{20}^{1}}{\cancel{2}}[2A + (19)(11)] = 10170$ (1)

$2A + 209 = 1017$

$2A = 808$

$A = 404$, where A is the first of the last 20 terms
(1)

$T_A = 8 + (N_A - 1)11 = 404$ (1)

$N_A - 1 = \frac{404 - 8}{11}$

$N_A = 37$ (1)

$k = 37 + 19$

$= 56$ (1)

$k =$ 56

(Total for Question 15 is 5 marks)

16 The first term of an arithmetic series is $(2t + 1)$ where $t > 0$
The nth term of this arithmetic series is $(14t – 5)$

The common difference of the series is 3

The sum of the first n terms of the series can be written as $p(qt – 1)^r$ where p, q and r are integers.

Find the value of p, the value of q and the value of r
Show clear algebraic working.

$a = 2t + 1$

$d = 3$

$T_n = 14t - 5 = 2t + 1 + (n-1)3$

$14t - 5 = 2t + 1 + 3n - 3$ (1)

$3n = 14t - 2t - 5 - 1 + 3$

$3n = 12t - 3$

$n = 4t - 1$ (1)

$S_n = \frac{4t-1}{2}\left[2(2t+1) + (4t-1-1)3\right]$ (1)

$= \frac{4t-1}{2}\left[4t + 2 + 12t - 6\right]$

$= 4t - 1\,(8t - 2)$

$= 32t^2 - 8t - 8t + 2$

$= 32t^2 - 16t + 2$

$= 2(16t^2 - 8t + 1)$

$= 2(4t - 1)^2$

$p = 2$, $q = 4$, $r = 2$ (1)

$p =$ 2 $q =$ 4 $r =$ 2

(Total for Question 16 is 4 marks)

17 Here are the first four terms of an arithmetic sequence.

38 −7 31 24 17

Find an expression, in terms of n, for the nth term of the sequence.

$a = 38$

$d = -7$

$T_n = 38 + (n-1)(-7)$

$= 38 - 7n + 7$

$= 45 - 7n$ (2)

$45 - 7n$

(Total for Question 17 is 2 marks)

18 Here are the first three terms of an arithmetic sequence.

$8p \qquad 7p - 3 \qquad 4p + 2$

The sum of the first n terms of the sequence is -1914

Work out the value of n
Show your working clearly.

$7p - 3 - 8p = 4p + 2 - 7p + 3$

$7p - 8p - 4p + 7p = 2 + 3 + 3$

$2p = 8$

$p = 4$ (1)

$a = 8(4) = 32$

$d = 7(4) - 3 - 32$

$= 28 - 3 - 32$ (1)

$= -7$

$S_n = -1914 = \frac{n}{2}\left[2(32) + (n-1)(-7)\right]$ (1)

$-1914 = \frac{n}{2}\left[64 - 7n + 7\right]$

$-3828 = n(71 - 7n)$

$7n^2 - 71n - 3828 = 0$ (1)

$(7n + 132)(n - 29) = 0$

$n = 29$ only since $n > 0$

$n =$ 29 (1)

(Total for Question 18 is 5 marks)

19 The sum of the first 80 terms of an arithmetic series, S, is 470

The 75th term of S is 14.5

The sum of the first X terms of S is 171

Work out the value of X
Show your working clearly.

$$S_{80} = \frac{80}{2}\left[2a + 79d\right] = 470$$

$$\therefore 40(2a + 79d) = 470 \quad (1)$$

$$2a + 79d = 11.75 \quad -(1)$$

$$T_{75} = a + 74d = 14.5 \quad (1)$$

$$2a + 148d = 29 \quad -(2)$$

$(2) - (1)$:

$$148d - 79d = 29 - 11.75$$

$$69d = 17.25 \quad (1)$$

$$d = 0.25$$

$$a = 14.5 - 0.25(74)$$

$$= -4 \quad (1)$$

$$171 = \frac{x}{2}\left[2(-4) + (x-1)0.25\right]$$

$$342 = x(-8 + 0.25x - 0.25) \quad (1)$$

$$342 = -8x + 0.25x^2 - 0.25x$$

$$0.25x^2 - 8.25x - 342 = 0$$

$$x^2 - 33x - 1368 = 0$$

$$(x - 57)(x + 24) = 0$$

X = 57 (1)

$X =$ 57

(Total for Question 19 is 6 marks)